

Validity

Derek Allen

A note for the instructor

I taught a two-semester reasoning course at the University of Toronto for many years. Enrolment was capped at 65. The course, open to students in all four undergraduate years, had no prerequisites. The first semester covered topics in informal logic, including argument recognition, argument analysis (by which I meant standardizing and diagramming arguments), and argument assessment. The second semester was devoted to ethical reasoning and legal reasoning.

I found that to develop enhanced reasoning skills students require repeated practice, so I assigned a number of exercises. Several of the exercises were mini-exercises worth 2% or 3% each. Others were longer exercises worth 7% each.

When introducing a new informal-logic topic, I used simple examples. More challenging examples followed in class, but for the most part I didn't rely solely on them for the purpose of preparing the students for the first-semester graded exercises. Instead, for each but one of these exercises I provided the students with a practice exercise that had been a graded exercise in a previous year. The students did the practice exercise outside class as part of their preparation for the corresponding graded exercise, and I posted answers and comments on the course website.

One of the graded exercises was a mini-exercise on assessing arguments for validity in natural/ordinary language. The term 'validity' was used in the course (and is used here) to mean *deductive* validity. Many instructors report that students often find validity a difficult concept to grasp, and this was my experience too. For this reason, I eventually decided to set *two* practice exercises for the mini-exercise on validity assessments, and to make the first of

them a teaching exercise with step-by-step instructions for doing the assessments; the second practice exercise was a graded exercise from a previous year. An example of a step-by-step teaching exercise on validity assessments appears below; answers are given separately.

The exercise requires familiarity with the following topics (which I had covered earlier in the semester):

- Arguments in one or another of the senses in which they are understood in logic textbooks; validity (understood as deductive validity); the idea of an argument's being valid or invalid "as it stands" (that is, without the addition of any tacit premises); assessing an argument for validity in ordinary language (as distinct from assessing an argument for validity by using procedures of formal logic); entailment; entails; implies.
- Necessary and sufficient conditions; "if and only if"; indicative conditional statements; the antecedent and consequent of an indicative conditional statement; an indicative conditional statement implies that the truth of its antecedent is sufficient for the truth of its consequent and that the truth of its consequent is necessary for the truth of its antecedent; affirming/denying the antecedent/consequent of an indicative conditional statement; *modus ponens*; *modus tollens*.
- The possibility that a two-premise conditional argument that denies the antecedent or affirms the consequent and is therefore formally invalid may nevertheless be a valid argument in virtue of its content; this will be the case if the argument's non-conditional premise entails the argument's conclusion.
- The use of a modality (e.g., 'probably', 'necessarily') in an argument to indicate, at least roughly, the degree of support that the arguer thinks the premises provide for the conclusion.

The above list may include topics that you won't have covered in your course before you cover validity, or that you won't cover at all. In this event, you would need to edit the following practice exercise accordingly. Or, for whatever reason(s), you might decide not to use even an edited version of the exercise. In this event, the exercise would still serve as an example of a step-by-step teaching exercise, and you might decide to devise such an exercise yourself (on validity or another topic) for use in your course.

The arguments in the exercise below are ones that I made up; they don't come from "real-world" contexts such as the media or politics or court judgments or online discussions of social issues. This is deliberate. The exercise is for students who have just been introduced to validity, and the point of the exercise is to help them learn how to assess arguments for validity in natural/ordinary language. For such students, it's best, I believe, to start with argu-

ments that have been devised specifically for this purpose and are free of the unclarity and complexities that often characterize arguments in “real-world” contexts. Once students have learned how to assess an assortment of such arguments for validity, they should be ready to try doing the same for selected “real-world” arguments.

Next, a comment about how the skill of assessing arguments for validity will have applications beyond the classroom. Whether an argument is a good one from a logical point of view depends (in part) on the nature of the connection between the argument’s premises and its conclusion. If the conclusion must be true if the premises are true, then the connection between the premises and the conclusion is the strongest possible: if the premises are true, their truth guarantees that the conclusion is true. If we are considering whether to “buy” some argument we encounter, it’s worth knowing whether the premises and conclusion are related in this way. If they are, then the argument is valid. Thus, students who become proficient at assessing an argument for validity, by determining whether the truth of its premises would guarantee the truth of its conclusion, acquire a useful logical skill—a skill they can apply to arguments they encounter in their daily lives beyond the classroom.

Finally, a transitional point. If the truth of an argument’s premises would guarantee the truth of the conclusion, it would be logically impossible for the argument to have both true premises and a false conclusion, and so the argument would satisfy the condition for (deductive) validity stated at the beginning of the following exercise, namely: an argument is valid (that is, deductively valid) if and only if it is logically impossible that it has both true premises and a false conclusion.

A Practice Exercise on Validity

Review

An argument is **valid** (that is, *deductively valid*) if and only if it is **logically impossible** that it has both true premises and a false conclusion. A valid argument can’t have the combination “true premises and false conclusion.”

An argument is **invalid** if and only if it is **logically possible** that it has both true premises and a false conclusion.

Recall that in this course *assessing an argument for validity* means ARGUING that the argument is valid or that it is invalid (NOT just asserting that it’s valid or that it’s invalid).

Assess for validity each of the following arguments as it stands (that is, without adding any tacit premises):

- (1) Figure 1 is square. Therefore, Figure 1 has exactly four sides.

Step 1: Identify the premise of the argument and the conclusion of the argument.

Step 2: Assume that the premise is true and ask yourself whether this assumption forces you to accept the conclusion as true.

Step 3: If you think that you must accept the conclusion as true if you assume that the premise is true, then you should conclude that the argument is **valid**. If you think that you are free to say that the conclusion is false even if you assume that the premise is true, then you should conclude that the argument is **invalid**.

Step 4: If you think that the argument is **valid**, how would you support (defend) this claim? If you think that the argument is **invalid**, how would you support (defend) this claim?

Step 5: Write out your validity assessment of the argument. Remember, this requires you to ARGUE that the argument is valid or that it's invalid; you can't just assert that it's valid or just assert that it's invalid.

For arguments (2)-(4), follow the same procedure as for argument (1).

- (2) Figure 1 is square. Therefore, Figure 1 is small.

- (3) Canada has more than nine provinces. Therefore, Canada has more than eight provinces.

- (4) Canada has 20 provinces. Therefore, Canada has more than 10 provinces.

Arguments (1)-(4) above each have one stated premise. The next four arguments each have two stated premises.

- (5) 1. Canada has 10 provinces.
2. The U.S.A. has 50 states.
Therefore,
3. Canada has one-fifth as many provinces as the U.S.A. has states.

Step 1: Identify the premises of the argument and the conclusion of the argument.

Step 2: Assume that the premises are true and ask yourself whether this assumption forces you to accept the conclusion as true.

Step 3: If you think that you must accept the conclusion as true if you assume that the premises are true, then you should conclude that the argument is **valid**. If you think that you are free to say that the conclusion is false even if you assume that the premises are true, then you should say that the argument is **invalid**.

Step 4: If you think that the argument is **valid**, how would you support (defend) this claim? If you think that the argument is **invalid**, how would you support (defend) this claim?

Step 5: Write out your validity assessment of the argument. Remember, this requires you to ARGUE that the argument is valid or that it's invalid; you can't just *assert* that it's valid or just *assert* that it's invalid.

For arguments (6)-(8), follow the same procedure as for argument (5).

- (6) 1. Canada has 13 provinces.
2. The U.S.A. has 52 states.
Therefore,
3. The U.S.A. has four times as many states as Canada has provinces.

- (7) 1. Canada has 20 provinces.
2. The U.S.A. has 80 states.
Therefore,
3. Canada has one-quarter as many provinces as the U.S.A. has states.

- (8) 1. The U.S.A. has 50 states.
2. Canada has 10 provinces.
Therefore,
3. The U.S.A. has a larger land area than Canada has.

For arguments (9)-(11), follow the same procedure as for argument (1).

- (9) 1. Jane plays several sports.
Therefore,
2. Jane likes sports.

- (10) 1. Bryan's girlfriend has broken up with him.
So it must be true that,
2. Bryan is sad.

- (11) 1. Amy has been sneezing all day.
This means that,
2. Amy has a cold.

For argument (12), follow the procedure set out below it.

- (12) 1. If Jack was at the party, then Jill was at the party.
2. Jack was at the party.
Therefore,
3. Jill was at the party.

Step 1: Underline the antecedent and the consequent of premise 1.

Step 2: Decide which of the following sentences is true: (a) Premise 1 implies that the truth of its antecedent is **sufficient** for the truth of its consequent. (b) Premise 1 implies that the truth of its antecedent is **necessary** for the truth of its consequent.

Step 3: Premise 2 either affirms or denies the antecedent of premise 1, or it affirms or denies the consequent of premise 1. Which of these four things does it do?

Step 4: Given your Step 2 answer and your Step 3 answer, would you say that the conclusion of the argument must be true if the premises are true, or would you say that the conclusion of the argument could be false even if the premises are true?

Step 5: If you think the conclusion of the argument must be true if the premises are true, then you should conclude that the argument is **valid**. If this is your conclusion, how would you support (defend) it? If you think the conclusion of the argument could be false even if the premises are true, then you should conclude that the argument is **invalid**. If this is your conclusion, how would you support (defend) it?

Step 6: Write out your validity assessment of the argument. Remember, this requires you to ARGUE that the argument is valid or that it's invalid; you can't just *assert* that it's valid or just *assert* that it's invalid.

For argument (13), follow the same procedure as for argument (12).

- (13) 1. If Jack was at the party, then Jill was at the party.
2. Jack wasn't at the party.
Therefore,
3. Jill wasn't at the party.

For argument (14), follow the procedure set out below it.

- (14) 1. If Jack was at the party, then Jill was at the party.
 2. Jill was at the party.
 Therefore,
 3. Jack was at the party.

Step 1: Underline the antecedent and the consequent of premise 1.

Step 2: Decide which of the following sentences is true: (a) Premise 1 implies that the truth of its consequent is **sufficient** for the truth of its antecedent. (b) Premise 1 implies that the truth of its consequent is **necessary** for the truth of its antecedent.

Step 3: Premise 2 either affirms or denies the antecedent of premise 1, or it affirms or denies the consequent of premise 1. Which of these four things does it do?

Step 4: Given your Step 2 answer and your Step 3 answer, would you say that the conclusion of the argument must be true if the premises are true, or would you say that the conclusion of the argument could be false even if the premises are true?

Step 5: If you think the conclusion of the argument must be true if the premises are true, then you should conclude that the argument is **valid**. If this is your conclusion, how would you support (defend) it? If you think that the conclusion of the argument could be false even if the premises are true, then you should conclude that the argument is **invalid**. If this is your conclusion, how would you support (defend) it?

Step 6: Write out your validity assessment of the argument. Remember, this requires you to ARGUE that the argument is valid or that it's invalid; you can't just *assert* that it's valid or just *assert* that it's invalid.

For argument (15), follow the same procedure as for argument (14).

- (15) 1. If Jack was at the party, then Jill was at the party.
 2. Jill was not at the party.
 Therefore,
 3. Jack was not at the party.

Answers for the Practice Exercise on Validity

- (1) 1. Figure 1 is square.
 Therefore,
 2. Figure 1 has exactly four sides.

Validity assessment: If the premise of this argument is true, the conclusion must also be true because by definition square figures have exactly four sides. Hence, the argument is valid as it stands.

- (2) 1. Figure 1 is square.
Therefore,
2. Figure 1 is small.

Validity assessment: Even if the premise of this argument is true, the conclusion may be false because square figures need not be small. Hence, the argument is invalid as it stands.

- (3) 1. Canada has more than nine provinces.
Therefore,
2. Canada has more than eight provinces.

Validity assessment: Nine is more than eight. Hence, if the premise of this argument is true (and it is true), then the conclusion must also be true. Thus, the argument is valid as it stands.

- (4) 1. Canada has 20 provinces.
Therefore,
2. Canada has more than 10 provinces.

Validity assessment: Twenty is more than ten. Hence, if the premise of this argument were true, the conclusion would have to be true as well. Thus, the argument is valid as it stands.

- (5) 1. Canada has 10 provinces.
2. The U.S.A. has 50 states.
Therefore,
3. Canada has one-fifth as many provinces as the U.S.A. has states.

Validity assessment: Ten is one-fifth of fifty. Thus, if the premises of this argument are true (and they are true), then the conclusion must be true as well. Hence, the argument is valid as it stands.

- (6) 1. Canada has 13 provinces.
2. The U.S.A. has 52 states.
Therefore,
3. The U.S.A. has four times as many states as Canada has provinces.

Validity assessment: Fifty-two is four times thirteen. Hence, if the premises of this argument were true, the conclusion would have to be true as well. Thus, the argument is valid as it stands.

- (7) 1. Canada has 20 provinces.
2. The U.S.A. has 80 states.
Therefore,
3. Canada has one-quarter as many provinces as the U.S.A. has states.

Validity assessment: Twenty is one-quarter of eighty. Hence, if the premises of this argument were true, the conclusion would have to be true as well. Thus, the argument is valid as it stands.

- (8) 1. The U.S.A. has 50 states.
2. Canada has 10 provinces.
Therefore,
3. The U.S.A. has a larger land area than Canada has.

Validity assessment: Even if the premises of this argument are true (and they are true), the conclusion could be false, for the 50 states of the U.S.A. could be contained within a total land area equal to or smaller than the total land area within which Canada's 10 provinces are contained. Thus, the argument is invalid as it stands.

Comment: According to a World Statistics website,¹ Canada has a larger land area than the U.S.A. has; if so, then argument (8) has a false conclusion. But it has true premises. Thus, if its conclusion is false, then it has true premises and a false conclusion and is therefore invalid. Why? Because a valid argument cannot have true premises and a false conclusion; rather, an argument with true premises and a false conclusion is *invalid*.

- (9) 1. Jane plays several sports.
Therefore,
2. Jane likes sports.

¹ www.mongabay.com/igapo/world_statistics_by_area.htm

Validity assessment: Even if it is true that Jane plays several sports, it may be false that she likes sports. It may be that she *hates* sports but plays several sports anyway because she is forced to do so by her parents. Thus, even if the argument's premise is true, the conclusion could be false, and so the argument is invalid as it stands.

- (10) 1. Bryan's girlfriend has broken up with him.
 So it must be true that,
 2. Bryan is sad.

Validity assessment: Even if Bryan's girlfriend has broken up with him, he may not be sad. It could be that he wanted to break up with her and that she discovered this and decided to break up with him in order to deny him the satisfaction of being the one to initiate the break-up. His pride is hurt, but overall he's happy, not sad, because he wanted their relationship to end, and he's glad it has. Thus, even if the premise of the argument is true, the conclusion could be false, and so the argument is invalid as it stands.

Comment: Note that this assessment involves telling a story in which the premise of the argument is true, the conclusion is false, and there's no contradiction (no talk of married bachelors, for example). Note too that the argument's modality ("must") is irrelevant to the validity assessment. It tells us that the arguer thinks the argument's premise provides very strong support for the conclusion; but this is irrelevant to the question of whether in fact (no matter what the arguer thinks) the conclusion must be true if the premise is true.

- (11) 1. Amy has been sneezing all day.
 This means that,
 2. Amy has a cold.

Validity assessment: Even if Amy has been sneezing all day, it could be false that she has a cold. It could be that she has a severe allergy (not a cold) and that this is why she has been sneezing all day. Thus, even if the premise is true, the conclusion could be false, and so the argument is invalid as it stands.

- (12) 1. If Jack was at the party, then Jill was at the party.
 2. Jack was at the party.
 Therefore,
 3. Jill was at the party.

Validity assessment 1: This argument affirms the antecedent and is therefore valid by *modus ponens*.

Validity assessment 2: If the conditional premise of this argument is true, then if its antecedent (“Jack was at the party”) is true this is sufficient for the truth of its consequent (“Jill was at the party”). If the argument’s non-conditional premise (“Jack was at the party”) is true, then the antecedent of the conditional premise is true. Hence, if both premises are true, the conclusion (“Jill was at the party”) must also be true, and so the argument is valid.

Comment: In a graded assignment, it wouldn’t be necessary to provide two validity assessments of the argument; one would be sufficient. The two assessments given here illustrate two different ways in which it can be argued that argument (12) is valid.

- (13) 1. If Jack was at the party, then Jill was at the party.
 2. Jack wasn’t at the party.
 Therefore,
 3. Jill wasn’t at the party.

Validity assessment: If the conditional premise of this argument is true, then if its antecedent (“Jack was at the party”) is true this is sufficient for the truth of its consequent (“Jill was at the party”). If the argument’s non-conditional premise (“Jack wasn’t at the party”) is true, then the antecedent of the conditional premise is false. But the consequent could be true even if the antecedent is false: Jill might have gone to the party because she heard that Jack wouldn’t be there and she wanted to avoid him. Thus, even if the premises of the argument are true, the conclusion could be false, and so the argument is invalid.

- (14) 1. If Jack was at the party, then Jill was at the party.
 2. Jill was at the party.
 Therefore,
 3. Jack was at the party.

Validity assessment 1: This argument affirms the consequent and is therefore invalid unless its non-conditional premise (2) entails the conclusion (3). But (2) does not entail (3). The situation could be this: Jill was at the party, but Jack decided to stay home and do some validity assessments. This situation shows that (3) could be false even if (2) is true; hence, it shows that (2) does not entail (3). Given this fact and the fact that the argument affirms the consequent, the argument is invalid.

Validity assessment 2: If the conditional premise of this argument is true, then the truth of the consequent (“Jill was at the party”) is *necessary* for the truth of the antecedent (“Jack was at the party”). But the truth of the consequent might not be *sufficient* for the truth of the antecedent, even if the conditional premise is true. It could be that:

- (a) Jack was at the party if and only if Jill *and* Vicky were at the party; and that
- (b) Jill was at the party, but
- (c) Vicky was not at the party.

In this scenario, Jack was not at the party because a necessary condition of his being there (namely, Vicky’s being there) wasn’t satisfied. Thus, the conclusion of the argument (“Jack was at the party”) could be false even if the premises are true, and so the argument is invalid.

Comment: As in the case of argument (12), it wouldn’t be necessary in a graded assignment to provide two validity assessments of the argument; one would be sufficient.

- (15) 1. If Jack was at the party, then Jill was at the party.
 2. Jill was not at the party.
 Therefore,
 3. Jack was not at the party.

Validity assessment 1: This argument denies the consequent and so is valid by *modus tollens*.

Validity assessment 2: If the conditional premise of this argument is true, then the truth of the consequent (“Jill was at the party”) is necessary for the truth of the antecedent (“Jack was at the party”). But if the non-conditional premise is true, then Jill wasn’t at the party. Hence, if both premises are true, then a necessary condition for Jack’s being at the party wasn’t satisfied, in which case he wasn’t at the party. Thus, if both premises are true, the conclusion must be true as well, and so the argument is valid.

Comment: As in the case of arguments (12) and (14), it wouldn’t be necessary in a graded assignment to provide two validity assessments of the argument; one would be sufficient.

About the author:

Derek Allen is Professor Emeritus of Philosophy at the University of Toronto. For many years he taught a course called Modes of Reasoning for which he wrote extensive class materials. During his career he received several teaching awards including a Leadership in Faculty Teaching Award for an "outstanding contribution to teaching excellence in Ontario" and a 3M Canada National Teaching Fellowship Award in recognition of "teaching excellence and educational leadership". He has written conference papers and commentaries in argumentation studies and has published journal and anthology articles in informal logic. He is a member of the editorial board of the journal *Informal Logic* and a past-president of the Association for Informal Logic and Critical Thinking. He and co-authors Sharon Bailin, Mark Battersby, and James B. Freeman wrote an invited article on critical thinking which has been published in the *Oxford Research Encyclopedia of Education*, Oxford University Press.