

---

## Logic and Critical Thinking

**G.C. Goddu**

### Introduction

This chapter is a primer on basic logical concepts that often appear in various critical thinking textbooks—concepts such as entailment, contraries, contradictories, necessary and sufficient conditions, etc. The chapter will not provide a historical genealogy of these concepts—in some sense critical thinking, argumentation theory, and formal logic all trace their roots back to at least Aristotle over two thousand years ago. As a result, for many of these concepts, determining whether the concept was a logic concept co-opted by critical thinking, or a critical thinking concept co-opted and changed by logic and then co-opted back again, is extremely difficult. Regardless, a brief orientation of the relationship of critical thinking and logic is in order.

Critical thinking, at least as it is most often justified, is a practical, skill-building exercise with the goal of improving our reasoning. This motivation, of understanding and improving our reasoning, has also been the motivation behind the development of logic over the past several thousand years. While we could study and understand each piece of reasoning individually, it is much more efficient to look for reasoning *patterns* that recur over and over again, to distinguish those patterns that are good from those that are bad, and so to find principles underpinning our reasoning that help us distinguish good reasoning from bad reasoning across the board. This push to generalize and theorize with the patterns of reasoning generated numerous formal logical systems, including the syllogistic and modal logics of Aristotle.

However, logic, especially formal logic, has not been constrained solely by the goal of understanding and improving our reasoning. Like abstract mathematics, the formal structures underpinning logical systems, are rich and complex enough to generate study all their own, with no concern for the original motivation that may have pushed us to study patterns of human reasoning.

Regardless, many of logic's concepts are still useful in organizing any study of reasoning.

In what follows I begin with a fairly substantial discussion of the core concept needed to understand the traditional logical concepts such as entailment or contradictory or necessary condition—the concept of a possibility. Once we have this notion in play, the definitions of the standard logical concepts, which I provide in Section 2, are quite straightforward. In the final section, I discuss the potential for misapplication of various concepts or distinctions.

## 1. Possibilities

### 1.1 Possibility and reasoning

The core concept of logic is the concept of a possibility (a case, a scenario, an option, a way things could be). While logicians and philosophers continue to work on illuminating the nature of possibilities, we can, even without a precise definition, still intuitively grasp the notion. You could stop reading right now or you could keep going. England didn't win, but England could have won, if they had scored their penalty kick. That die, when rolled, will land on one of six possible sides. There are many things that might happen if the bill is passed into law. According to 18th century philosopher Gottfried Leibniz, God surveyed all the possible ways the universe might be and, being omnibenevolent, chose to create the best one (this one!?).

We appeal to possibilities all the time in our reasoning. Indeed, if there were but one way things could be and we knew completely what that way was like, then we would not need to reason at all—we would just know how things were going to unfold. But given that (i) we do not know completely how things are or how the future is going to unfold and (ii) we assume there are multiple possibilities for how the future might unfold, we need to reason about the ways things could be in order to learn how things are and how to best manage whatever the future brings. For example, the detective investigating a suspicious death gets a new piece of evidence—the deceased was killed by a rare poison. As a result, some scenarios are closed off as viable explanations of the death—e.g., the deceased was deliberately killed by someone who did not have access to the poison. Other scenarios, ones that may not have been in the detective's awareness before the new piece of evidence was acquired, become relevant—e.g., that someone who knew or at least had access to the poison was responsible for the death. As a result, a new line of inquiry opens for the detective: find out who had access to the poison. Similarly, a doctor runs a series of tests to try to eliminate certain possible explanations for a given patient's symptoms. Given certain results the possible explanations get narrowed down to one (and hopefully a treatment is

available); given other results multiple possibilities remain and the doctor has to decide which tests may be required for progress to be made; unexpected results, while eliminating some possibilities may open up new possibilities that the doctor had not originally been considering. Finally, you are trying to decide when and in which order to run a list of errands. You take into account the likely lines at each location at different times of day, and the likely traffic at different times of day. After evaluating the possibilities, you choose the best option for you.

## *1.2 Types of possibilities*

### *1.2.1 Physical and epistemic possibilities*

Given the ubiquity of possibilities in our reasoning, theorists often classify the possibilities. For example, physicists are interested in distinguishing the physical possibilities (the possibilities consistent with the laws of physics) from the physical impossibilities (the possibilities inconsistent with the laws of physics). Other general types of possibilities include epistemic possibilities—scenarios consistent with what we know; moral possibilities—those consistent with a given moral code; legal possibilities—situations consistent with what is permitted by a given legal code. We can even combine these types—epistemic physical possibilities are those that are consistent with the laws of physics as we currently know them. If what we know about the world changes at a fundamental level, what once was epistemically physically possible (measuring time independently of motion or gravity) may become epistemically physically impossible. Like for the detective and the doctor above, new, unexpected evidence may require an adjustment by the scientist in what possibilities are under consideration as viable explanations.

### *1.2.2 Equally probable possibilities*

Two other sorts of possibilities deserve mention. Probabilistic reasoning depends on possibilities of a very special sort—equally probable possibilities. To determine the probability that a fair coin will come up heads we assume that there are two equally likely possibilities, “heads” and “tails” (we usually ignore the extremely unlikely, though still physically possible situation in which the coin lands and stays on its edge). Failing to consider the relevant equally likely possibilities can make our probabilistic reasoning go awry. You will either win the lottery or you will not. There are two possibilities here, but treating them as equally likely is certainly an obvious mistake. Assuming the lottery is fair, the relevant equally likely possibilities are that each individual ticket (or set of numbers) will be the winner. If your ticket is one of many, then the probability you will win the lottery is much lower than the probabil-

ity of your losing. Less obvious, but equally problematic is the following sort of case:

Three drawers contain the following mixture of coins—one contains two gold coins, one contains two silver coins, and one contains one gold coin and one silver coin. Without looking you pick a drawer, open it, and pick out a coin. When you open your eyes, you see the coin is gold. What is the probability that the other coin in that drawer is gold?

Many will reason as follows. The coin came from either the gold/gold drawer or the gold/silver drawer. Each drawer is equally likely and if it came from the gold/gold drawer the other coin is gold. But if it came from the gold/silver drawer the other coin is silver. Hence, the probability the other coin is gold is  $\frac{1}{2}$  or 50%. Unfortunately, the two possible drawers are not the relevant equally likely possibilities (no more than your winning or losing were the relevant equally likely possibilities in the lottery case). The relevant possibilities are opening a drawer and picking out a coin without looking. There are six different equally likely ways that could happen, one for each coin. Once you gain the new evidence that the coin you picked is gold when you open your eyes, you can eliminate three of the six possibilities, i.e. the ones in which you pick a silver coin. Of the three possibilities left two are such that the other coin is gold, i.e., the two possibilities in which you pick one of the two coins from the gold/gold drawer. Only in the gold/silver case is the other coin silver. Hence, the probability of the other coin being gold is  $\frac{2}{3}$ . The moral here is that accurate probabilistic reasoning requires identifying and using the relevant equally likely possibilities from amongst all the sorts of possibilities that may present themselves—not always an easy task.

### *1.2.3 Practical possibilities*

Another significant type of possibility, especially in our everyday reasoning, is practical possibility—possibilities that are consistent with our means, desires, and will (or perhaps our epistemic practical possibilities—the possibilities that, given what we know or believe, are consistent with our means, desires, and will). When deciding how to get to an important meeting across town you are likely to not even consider the possibility that you flap your arms and fly, or the possibility that you use your personal matter/energy transport device, or even the possibility that you sprint all the way there. The first is physically impossible; the second, while perhaps physically possible, is beyond our current technological means; and the third, while certainly physically possible, is quite likely beyond your will and most certainly contrary to your strong desire to not arrive at the important meeting sweating profusely and gasping for breath. Instead you consider what your actual transportation

options are (your own car, Uber, taxi, walk, subway, or some combination), how much time you have, how much money you are willing to spend, and then you try to find the optimal possibility (usually constrained by the desire to not spend too much time actually calculating the optimal possibility). Mundane decisions about which possibility to actualize like this happen all the time: what to eat this week, which movie to go see, what to do after dinner, when to get your hair cut, etc. Though mundane, they are still of interest to critical thinking or argumentation theorists since businesses and advertisers spend billions of dollars and devote millions of work-hours to trying to influence your desires and will in order to persuade you to choose their product.

Of more social significance are your individual choices that impact larger groups—in particular (if you live in a democracy) your voting choices, your decisions about how much effort you put into monitoring the outcome of your voting choices, and what the individuals or policies you voted for end up doing. In an optimal world, your political representatives would enact policies that benefit the most people in the most cost efficient, affordable, and just way. Of course, there may be little agreement about what is the most affordable, or just, or beneficial option, especially if what elected representatives take to be the best option is what will get them re-elected rather than what is actually good for their constituents. Regardless of the complexities and intricacies of public policy debate and decision-making, at the core is an attempt to find and agree upon a practical possibility, from amongst the myriad available, to actualize for our mutual benefit.

Given so many types of overlapping sets of possibilities, many of which differ for different individuals or groups of individuals—your set of practical possibilities does not likely match that of your neighbor even if the two sets overlap significantly; compare your set with someone of quite different socio-economic means and the sets overlap even less—and it is no surprise that numerous problems can arise when reasoning with and about possibilities. Individuals can consider too many possibilities, or more commonly, fail to consider all the relevant possibilities. For example, human beings are quite prone to confirmation bias—taking confirming instances as justifying an already-accepted theory or explanation rather than actively seeking out or testing for disconfirming instances. Detectives, or doctors, or researchers can become so fixated on the explanation they already believe to be correct that they are blind to the alternate explanations that are still consistent with the evidence available. In the case of probabilistic reasoning, we already saw cases of considering the wrong set of possibilities. Reasoners can also illegitimately shift the set of possibilities under consideration or shift the value assigned to various possibilities mid-reasoning. An egregious example can occur in public policy debates over the negative consequences of potential

policies. When negative consequence X is a potential consequence of the opposition's preferred policy it is judged to be likely enough to count as a reason against the policy, but when negative consequence X is a potential consequence of one's own preferred policy, it is judged not to be likely enough to count as a reason against the policy. Identifying the correct set of possibilities and correct relative values of those possibilities is essential to reasoning correctly in numerous situations and yet identifying and ranking possibilities is often an extremely difficult task.

#### 1.2.4 Logical possibilities

One way to try to sidestep some of these problems is to determine what reasoning holds no matter what the possibilities in question are—to determine the patterns of reasoning that work in *all* the possibilities. After all, if a piece of reasoning works no matter what possibility you are considering, then you do not need to worry whether you are considering the right set of possibilities or not. Hence, one goal of formal logic is to be able to identify the structure that defines *all* the ways things could be, i.e., the logical possibilities.

The rough and ready notion of a “logical possibility” is a possibility that has no contradiction in it. Whilst it is not logically possible for an individual to both exist at a particular time and place and not exist at that time and place, which is contradictory, it is logically possible that the person exist in Montana in one instant, and then exist on one of the moons of Jupiter, say Io, in the next. There is no contradiction in the possibility that you exist in Montana in one instant and on Io in the next. But this possibility, while logically possible, is not physically possible. Given the distance from Montana to Io, we would need to violate the physical restriction on moving matter or energy (currently travelling below the speed of light) faster than the speed of light to get from Montana to Io from one instant to the next, so such travel is physically impossible.

Earlier I said that philosophers are still investigating and debating the nature of possibilities. But, whatever they are, there is one actualized one and lots of unactualized ones. In Leibniz's argument that this world is the best of all possibilities, God examines all the possibilities and then actualizes the best one. Even if you doubt Leibniz's argument, of all the myriad ways this universe could be, it is in fact one way, namely, the possibility that is actualized. The detective has numerous possibilities in mind about who is responsible for the deceased's death; the detective hopes that by finding more evidence the possibilities can be reduced to one, the actual one. When you are deciding what to do tomorrow, you consider numerous possibilities and then engage in actions that make one (hopefully the one you wanted) actual.

But since there are lots of unactualized possibilities and only one actual possibility, how do we distinguish the unactualized possibilities from each other? Quite simply by what is true and false at each possibility. I flip a coin twice. There are four possible outcomes. Heads for the first flip and heads for the second; heads for the first and tails for the second; tails for the first, and heads for the second; and tails for both. Suppose the coin comes up tails on the first and heads on the second—that is the possibility that got actualized. How do we distinguish the three non-actualized possibilities? Well, in the first and second it is true that the coin first came up heads, but in fourth it is false that the coin first came up heads. But possibilities one and two differ in what is true and false of the second coin flip.

### *1.3 Declarative sentences and propositions*

Given that we distinguish possibilities by what is true and false if they are actualized, one proposal for understanding possibilities is just as sets of declarative sentences. For example, the first coin flip possibility would be the set {"the first flip of the coin came up heads", "the second flip of the coin came up heads"}. While initially appealing, the problem with this proposal is that sentences are not as well behaved as is needed to demarcate possibilities. Why?

Sometimes different sentences describe the same possibility or state of affairs. For example, "George is a bachelor" and "George is an unmarried male of marriageable age" describe the same state of affairs, but are different sentences since they are composed of different words. But since they are different sentences, sets that differ only in regards to which of these two sentences they contain are still different sets, and so different possibilities. Yet, we agreed the sentences were just two different ways of talking about the same possibility.

Alternatively, sometimes the same sentence can be used in different ways to describe different possibilities. For example, the sentence "The movie was a bomb" used in the United States likely describes a state of affairs in which the movie was bad, but the same sentence used in the United Kingdom likely describes a state of affairs in which the movie was good. But if one sentence can be used in different ways to describe different possibilities, then, once again, we cannot identify possibilities merely with sets of sentences.

To avoid the vagaries of sentences, logicians usually resort to propositions—what it is that declarative sentences express. "George is a bachelor" and "George is an unmarried male of marriageable age" express the same proposition about George's marital status. "England won the World Cup in 1966" expresses a true proposition about the English national soccer team; " $2 + 2 = 5$ " expresses a false proposition about the sum of 2 and 2. We use

declarative sentences to express propositions directly, but other language use often involves them. For instance, when we ask, “did Hungary win the World Cup in 1938?” we wonder whether the proposition that Hungary won the 1938 World Cup is true or false. If we get the correct answer (they did not win—they lost to Italy 4-2 in the finals), then we stop wondering whether it is true or false and start believing it is false (and if the *belief* is strong enough and acquired in the correct way, we might even *know* that the proposition is false).

Instead of treating possibilities as sets of sentences, many logicians treat possibilities (or at least model possibilities) as sets of propositions. There are technical details that might require modifying even this proposal, but since the resolution of these details is unlikely to be relevant to the critical thinking project, we can take possibilities to be sets of propositions. The propositions that are members of a particular possibility are said to be true or obtain at that possibility. Propositions that are not members of a particular possibility are false at that possibility or do not obtain at that possibility. Armed with the concepts of (i) a possibility and (ii) propositions being true at or obtaining at possibilities, we can define many of the logical concepts that pervade logic and critical thinking textbooks. So even though some of the logical concepts that are forthcoming are, in some textbooks, defined in terms of sentences, the more common way is to define them in terms of propositions.

## 2. Logical concepts

### 2.1 Types of propositions

I begin by discussing some common types of propositions that arise in our reasoning. The most basic is a *simple* proposition, propositions expressed by such declarative sentences as “George is a bachelor” or “the sky is blue” or “Romeo loves Juliet.” Simple propositions attribute something to some object(s) or thing(s). In the first case, of George, that he is a bachelor, and in the third case, of Romeo, that he loves another object, namely Juliet. *Negations* of simple propositions, propositions expressed by such declarative sentences as “George is not a bachelor” or “Hungary did not win the 1938 World Cup” say that the simple proposition does not obtain. Of course, we do not speak declaratively solely by affirming either simple propositions or the denial of simple propositions; we combine or modify our simple propositions such as in:

- (1) “George is a bachelor, and so is Todd”;
- (2) “Mary loves Antonio, but he does not love her back”;
- (3) “George went to Sophie’s house or he went to the movies”;



- (4) “If the butler did not do it, then the cook did”;
- (5) “If I take the subway, I will be on time for my meeting”;
- (6) “Every student in this class is eligible”;
- (7) “Someone deliberately killed the deceased”;
- (8) “In order to be on time for your meeting, you must take the subway”;
- (9) “England did not win the game, but they might have if they had scored their penalty kick in the last minute.”

The first two sentences express *conjunctions*. For a conjunction to be true, both sub-parts of the conjunction have to be true. So for “George is a bachelor *and* so is Todd” to be true, both “George is a bachelor” and “Todd is a bachelor” must be true. [For ease of exposition I will often omit the phrase “the proposition expressed by” before mentioning sentences as I just did above.]

The sentence “George went to Sophie’s house *or* he went to the movies” expresses a disjunction. There are two sorts of disjunctions—inclusive and exclusive. For an *inclusive disjunction* to be true, at least one of the sub-parts must be true. For an *exclusive disjunction* to be true, exactly one of the sub-parts must be true. If our sentence about George expresses an exclusive disjunction, then for it to be true George needs to be in exactly one of two places—at Sophie’s house or at the movies. This is likely to be the usage of someone trying to tell us where George is at a particular moment. If, on the other hand, the sentence expresses an inclusive disjunction, then it will be true if George went to one of those locations and is still true if George went to both. This is likely to be the usage of someone just trying to lay out where George might have gone over a period of time. While some languages have different words for expressing inclusive and exclusive disjunctions. English relies on context or background knowledge, sometimes with limited success, to try to distinguish which type of disjunction is being expressed. Legal documents, in order to avoid the ambiguity of ‘or’ in English, often spell out exclusive disjunctions as “A or B and not both A and B” while representing inclusive disjunctions as “A and/or B”.

Sentences such as: “*If* the butler did not do it, *then* the cook did” and “*If* I take the subway, [*then*] I will be on time for my meeting,” express conditional propositions. Conditionals are frequently used in natural languages such as English, yet there is little agreement on how they are to be analyzed logically. (Some theorists even go so far as to deny that conditional sentences express propositions at all.) Usually the disagreement concerns determining exactly what it takes for conditionals to be true, but there is widespread agreement that declarative conditionals are false if the ‘if’-part, the *antecedent*, is true, and the ‘then’-part, the *consequent*, is false. If it is true that I take the subway, and false that I will be on time for my meeting, then the conditional “If I take

the subway, I will be on time for my meeting,” is false. As a consequence, logic has defined a minimal version of the conditional, called the *material conditional*. *Material conditionals* are false if the antecedent is true and the consequent is false, but true otherwise—in other words, material conditionals are the most permissive when considering what it takes for a conditional to be true. There has been much debate about whether *indicative conditionals* such as “If the butler did not do it, then the cook did” just express material conditionals or rather express something stronger. Despite the disagreement, the most common articulation of conditionals in introductory logic texts is in terms of material conditionals, and it is most often this sort of conditional that is co-opted into critical thinking texts. One merely needs to keep in mind that the work on understanding conditionals is far from finished.

“Every student in this class is eligible” expresses a *universal proposition*—a proposition that attributes something to every member of a specified group. For a universal proposition to be true there can be no instance of a member of the group not having the specified attribute. If “Every student in this class is eligible” is true, then there is no student in the class who is not eligible. Oftentimes the group is not fully identified in the sentence used to express the proposition. For example, saying “All the beer is in the fridge” or “All horses have heads” are unlikely to be taken as expressing that every single beer in the universe is in a particular fridge or that there is no single instance of a headless horse anywhere. Depending on the context of use, likely plausible interpretations of those sentences would be: “All the beer we brought home from the store (and which has not already been drunk) is in the fridge” and “Typical, normal live horses have heads.” But once the group is fully specified, for a universal proposition to be true, every member of the group must have the attributed property or properties.

Instead of saying that everything in a given group has a stated attribute, we often merely want to convey that *at least one* thing or *some* things in the group have a particular property as in “Someone deliberately killed the deceased.” Such propositions are *existential* propositions. They are true when at least one object in a specified group has a specified attribute. For example, “Some student is eligible” is true just so long as at least one student is indeed eligible.

So far, most of our examples of propositions can be true or false given a single possibility. Suppose we restrict ourselves to just the actual possibility—then it is either true that George is a bachelor at the actual possibility or it is not; if, at the actual possibility, there is no student in the class who is not eligible, then the universal “Every student in class is eligible” is true at the actual possibility and otherwise false. But some of our declarative sentences are not just about one possibility; rather, they depend on multiple possibili-

ties. Sentences such as the last two on our list, which express *modal* propositions are examples. (They are called “modal” because expressions such as “must”, “can”, “might”, “would”, etc., were said to indicate the “mode” of the component proposition.)

Different modal expressions have different truth conditions. Consider, for example, the sentence—“In order to be on time for your meeting you must take the subway.” For it to be true, all the possible ways (probably some set of practical possibilities constrained by the background in which the sentence is uttered) in which you make the meeting on time include your taking the subway. In the case of England losing, but winning if they’d scored their penalty, the first part is a negation that is true just so long as England won is false. So the first part tells us what the actual possibility is like. But the second part tells us what the relevantly similar possibilities except for England scoring their penalty, are like—namely, that England won in at least one of those possibilities. Compare that with the stronger claim that England would have won if they had scored their penalty—that claim will be true just so long as England wins in all the relevantly similar possibilities. (Part of the debate about conditionals is whether even conditionals without explicit modal terms, such as ‘might’ or ‘would’ or ‘must’, etc. are really expressing propositions concerning multiple possibilities, and not just the actual one—again, a debate I will not be able to resolve here.)

This list is not at all meant to be exhaustive of the type of propositions we express via our declarative sentences. Rather, it is meant to give a flavor for the sorts of propositions dealt with in first and second logic courses, the sorts of propositions that logicians attempt to model and define clearly and precisely in their basic systems. Why are logicians interested in these sorts of propositions? Because they show up in many of the reasoning patterns that we use over and over. For example, if I tell you George is either at Sophie’s or at the movies, and you tell me he is not at movies, we both hopefully reason that we should check for George at Sophie’s house. Another example: If the US Internal Revenue Service says that all US taxpayers satisfying their three specified conditions can claim a particular deduction, and you’re a US taxpayer who determines that you satisfy those three conditions, you should reason that you can take that particular deduction. It is by recognizing these types of propositions, and the patterns that result in combining them, that formal logic, which focuses on the patterns, gets its impetus. But regardless of whether one is focusing on the goodness of patterns or more generally on the patterns and content of reasoning, both critical thinking theorists and logicians need to take special care in determining what proposition a given sentence in a particular context expresses, for without understanding the correct

proposition we will not be considering and evaluating the correct possibilities.

Even though understanding and classifying what propositions various declarative sentences express is an ongoing project, there is another classification scheme that logicians often appeal to—necessary truths (also called tautologies), necessary falsehoods (also called contradictions), and contingent propositions. The definitions are as follows:

*Necessary Truth:* A proposition that is true in all possibilities.

*Necessary Falsehood:* A proposition that is false in all possibilities.

*Contingent Proposition:* A proposition that is true in some, but not all, possibilities.

Sentences such as: “Either Socrates corrupts the youth of Athens or he does not”, or “If it is raining, then it is raining” express necessary truths. For every possibility there is, either Socrates corrupts the youth of Athens in that possibility or he does not. Some have wondered if there any non-trivial tautologies, since the standard examples, such as the ones I just gave, seem to be pretty trivial, uninformative sentences. Many theoreticians hold that the truths of mathematics are all necessary truths and many of those truths are certainly non-trivial—they often take a lot of work for us to know that they are true. Others point out that even if many necessary truths seem trivial or uninformative, they are still very useful. Plato, for example, uses the Socrates sentence in part of his dialogue concerning whether Socrates should have been found guilty of a particular offense. Plato starts with the obvious truth that either Socrates corrupts the youth or he does not, but proceeds to argue that in either case, Socrates should not be found guilty.

Sentences such as “At a particular moment in time, Socrates is over six-feet tall and Socrates is not over six-feet tall” express necessary falsehoods. For any possibility, and any moment of time in that possibility, Socrates cannot be both over six-feet tall and not over six-feet tall. Necessary falsehoods, or *contradictions* as they are more commonly called, are useful as sign-posts of something having gone drastically wrong in our reasoning. If we can show that someone’s position contains or leads to a contradiction, then we show that they aren’t even talking about a genuine possibility at all, but rather an impossibility. Good reasoners generally want to avoid being committed to impossibilities, so they try to avoid being committed to contradictions in their reasoning.

Most of the propositions we deal with in our everyday reasoning are contingent ones. “The coin landed heads on the first flip” is true in some possibilities, but false in others. “George will arrive on time” is true in some, but false in others. Even complex propositions, such as “If I take the subway, I will make it to the meeting on time” are likely to be true in some possibilities

(smooth running reliable subway system) and false in others (an unreliable or scanty subway system). The challenge for good reasoners, of course, is to try to figure out, on the basis of what we already know, and the acquisition of new evidence, which propositions are in fact true at the actual possibility and which are not true. The detective, the doctor, the scientist, the everyday reasoner, are all reasoning using various possibilities in order to try to determine which propositions are true or false at the actual possibility.

## *2.2 Relations amongst propositions*

Given that reasoning is the moving from given propositions to other propositions, and logicians are trying to understand correct reasoning, many of the important concepts of logic concern not just types of propositions, but the relations amongst propositions. I finish this section with definitions, examples, and discussion of eight such relations.

### *Necessary condition*

*One proposition, A, is a necessary condition for another proposition B, if there is no possibility in which B obtains and A does not.*

If A is a necessary condition for B, then you cannot have B without A. For example, if it is true that meeting the eligibility requirements is a necessary condition for legitimately holding office, then there is no possibility in which one legitimately holds office and does not meet the eligibility requirements. But if it is false that meeting the eligibility requirements is a necessary condition for legitimately holding office, then there is at least one possibility in which one legitimately holds office and does not meet the eligibility requirements.

### *Sufficient condition*

*One proposition, A, is a sufficient condition for another proposition B, if there is no possibility in which A obtains and B does not.*

If A is a sufficient condition for B, then A guarantees B. For example, if it is true that getting a perfect score on every assessment is sufficient for passing the course, then there is no possibility in which one gets a perfect score on every assessment and one does not pass the course. If it is false, then there is at least one possibility in which one gets a perfect score on every assessment and still does not pass the course.

In many elementary logic or critical thinking textbooks, necessary and sufficient conditions are treated as material conditionals. For example, “George attending class is sufficient for George passing the course” is treated as “If George attends class, then George passes the course.” But necessary and suf-

ficient conditions cannot be material conditionals, since denying a sufficient or necessary condition is not the same as denying a material conditional. For example, saying “George attending class is not sufficient for George passing” is not the same as denying the material conditional “If George attends class, then George passes the course” is true. Denying the material conditional is just saying that it is actually the case that George attends class, but does not pass the course, i.e., that the antecedent is true and the consequent is false. But denying that George’s attending is sufficient for George’s passing is not saying that George attends and does not pass, but rather says that there is a possibility, not necessarily the actual one, in which George attends, but does not pass. In other words, necessary and sufficient conditions are describing what is true of a range of possibilities.

### *Equivalence*

*Two propositions are equivalent just so long as there is no possibility in which one is true and the other is false.*

In other words, for each possibility, the two propositions are either both true or both false. For example, “All Euclidean triangles have three sides” and “All Euclidean triangles have three interior angles” are both true in all possibilities and false in none, so they are logically equivalent to each other. Similarly, “Either Peter failed to make the team or Abigail failed to make the team” is logically equivalent to “Abigail and Peter did not both make the team.” If the first proposition is true, then, on an inclusive disjunction reading, at least one of the two did not make the team, so it is true that they did not both make the team. If on the other hand the first proposition is false, then it is false Peter failed to make the team (and so made it) and it is false Abigail failed to make the team (and so also made it), in which case both made the team and the second proposition is also false. Since the propositions are true in the same possibilities and false in the same possibilities they are logically equivalent.

Equivalence of proposition is not to be confused with the equivalence of sentences. Two sentences are equivalent, such as “George is a bachelor” and “George is a unmarried male of marriageable age” just in case they express the same proposition. Two distinct propositions, on the other hand, are equivalent just in case they are true or false in exactly the same possibilities. Of course, without a clear notion of the identity conditions of propositions, it is often hard to determine whether we have two sentences expressing one proposition, or two sentences expressing two distinct propositions that are equivalent to each other. [Like possibilities, theorists are still debating how to understand propositions. For example, here I have defined possibilities as sets of propositions, but some theorists reverse the order of dependence and

define propositions as sets of possibilities, i.e., the possibilities at which they are true. Either way, having defined one concept in the terms of the other, the theorist still owes us an account of the undefined concept—a task theorists continue to pursue.]

### *Consistency*

*Two propositions are consistent with each other just in case there is at least one possibility in which both are true.*

For example, “Sphere A is completely red” is consistent with “Cube B is completely blue” just so long as there is a possibility in which both are true. But “Sphere A is completely red” is inconsistent with “Sphere A is partly blue” since there is no possibility in which both are true.

### *Contrary*

*Two propositions are contrary to each other if there is no possibility in which both are true.*

Contrariness is a kind of inconsistency. As we just saw, “Sphere A is completely red” is inconsistent with “Sphere A is partly blue” because there is no possibility in which both are true, i.e., because they are contrary to each other. But even though both propositions cannot be true together, they both could be false together, such as in possibilities in which “Sphere A is completely green” is true. But there is an even stronger kind of inconsistency, than mere contrariness.

### *Contradictory*

*Two propositions are contradictory to each other if there is no possibility in which both are true or both are false.*

“Sphere A is completely red” is contradictory to “Sphere A is not completely red” since if one is true, the other is false and if one is false, the other is true. Similarly, if it is true that “Snow guarantees skiing” then it is false that “There is a possibility in which there is snow and no skiing” and vice versa.

One important reason to keep these two kinds of inconsistency separate is that reasoners sometimes treat inconsistency as if it were just the same as being contradictory—they reason that if two states of affairs are inconsistent, then if one is false, the other one must be true. But such reasoners miss or ignore the possibility that two inconsistent propositions might still both be false, and as we saw in the previous section, ignoring or missing relevant possibilities is prone to generate reasoning errors. Hence, knowing whether two propositions are consistent, or contrary, or contradictory gives us important

information about which possibilities are still relevant to whatever inquiry or reasoning we are pursuing using those propositions.

Since logicians are motivated by the goal of distinguishing good reasoning from bad reasoning and at least one part of good reasoning is that what we reason from adequately supports what we reason to, logicians are very interested in relations of adequate support. One very special kind of adequate support is entailment.

### *Entailment*

*Proposition A entails proposition B just so long as there is no possibility in which A is true and B is false.*

For example, “Sam’s car weighs over 1000kg” entails “Sam’s car weighs at least 500kg”—any possibility in which Sam’s car is over 1000kg it is clearly at least 500kg. “Sam’s car is a red hatchback” entails “Sam’s car is red” and “Sam possesses a car” and “Sam’s car is a hatchback”. Instead of talking about what a single proposition entails, logicians are often interested in what a group or set of propositions entails. [A set of propositions entails another proposition just so long as there is no possibility in which all members of the set are true and the other proposition is false.] For example, “George went to Sophie’s house or to the movies” and “George did not go to the movies” entail “George went to Sophie’s house.” On the other hand, “If Sally attends class, then she passes the course” and “Sally passes the course” does not entail “Sally attends class,” since there are possibilities in which Sally can study well enough on her own and there is no attendance requirement, such that while it is true that “If Sally attends, then she passes the course” and true that “she passes the course”, it is false that “she attends class”.

Logic, especially formal logic, is primarily interested in entailment and other consequence relations. But at the elementary levels of logic at least the concept of entailment is applied to a concept that is also of interest to critical thinking and argumentation theorists—the concept of an argument. In logic, arguments are often modeled as a set of a set of propositions (the premises) and another proposition (the conclusion). [See Chapters 8 and 9 of this volume for a more detailed discussion of the concept of an argument.] Logicians define *validity*, a property of arguments, in terms of whether or not the entailment relation holds between the premises and the other proposition, the conclusion. If the premises entail the conclusion, then the argument is valid, i.e., there is no possibility in which the premises are true and the conclusion false, and otherwise the argument is invalid. [Validity here is not to be confused with the notion of ‘valid’ that is used in everyday speech to signify that something is “good” or “worthy of further consideration”, as in: “She made a valid point, when she said ....”. Nor is it to be confused with the notion of ‘valid’



that is used in survey research to signify the goodness or utility of a measuring instrument or the results of such an instrument—for that concept see Chapter 19 of this volume.]

In the previous section, I said that one of the motivations for studying logic was to try to find properties of good reasoning that would hold in all the possibilities. Entailment (and so validity) is one such property. If the arguments you make are valid, i.e. if your reasons entail your conclusion, then your reasoning, at least in terms of support, is good reasoning. Of course, other aspects of that reasoning might be problematic, but at least you know that your reasons, if true, guarantee your conclusion, no matter what set of possibilities is the relevant set.

But consider: Most of the coins on the table are heads-up and that quarter is a coin on the table, so it is heads up. “Most of the coins on the table are heads-up” and “That quarter is a coin on the table” do not entail that “That quarter is heads up” and yet in many situations we would likely say that the first two propositions give very strong reasons to believe the third. In other words while entailment is a sure sign of inferential goodness in reasoning, the lack of entailment does not necessarily mean there is a lack of inferential goodness. Sometimes we say our reasoning is good enough, even if our reasons do not entail what we infer from them. If, in the possibilities in which our reasons are true, enough of them also have what we infer to be true, then we can say that the inferential link is good because the reasons sufficiently support our conclusion. The general definition of sufficient support is as follows:

#### *Sufficient Support*

*Proposition A (or a set of propositions) sufficiently supports a proposition B just so long as, in enough of the possibilities in which A (or the set of propositions) is true, B is also true.*

What counts as “enough” often varies from context to context. For example, in civil litigation, the conclusion of wrongdoing has to be supported by a preponderance of the evidence, i.e., the possibilities in which the defendant did what they are accused of, should be the case in more than 50% of the possibilities in which the provided evidence is true. But in criminal cases, the conclusion of wrongdoing should be supported beyond a reasonable doubt (which, at least if we take the vast majority of judges’ views on what that means, is above 80%). Statistical significance for supporting various hypotheses in the sciences is often set at 95% or higher. Determining what should count as “enough” in various contexts is often extremely challenging. At the very least, some of what counts as “enough” depends on the importance of the outcome. For example, since criminal sanctions are so much higher than civil

sanctions, we demand more assurance that the evidence supports the conclusion of wrongdoing in the criminal case than in the civil case.

Logicians, I said, are primarily interested in consequence relations such as entailment. Different types of logic study these relations in different domains. For example, temporal or tense logics are interested in determining the consequence relations amongst uses of temporal phrases, such as, “in the future”, “in the past” and “now”. Modal logics study the consequence relations amongst propositions containing modal terms such as “must”, “can”, etc. But in addition to distinguishing types of logics by the types of propositions being modeled, logics are also categorized in terms of the type of consequence relation being studied. At the most general level, there are two types of logic—deductive and inductive. Deductive logic is concerned with entailment. Inductive logic is concerned with consequence relations weaker than entailment. Unsurprisingly, since there are many consequence relations weaker than entailment, inductive logic is a much less unified field of study than deductive logic. As we shall see in the next section, there are other uses of the terms ‘deductive’ and ‘inductive’, but these are generally misuses—the key difference between inductive and deductive logic is the type of consequence relation being studied.

I conclude this section with a final point about these eight definitions. They have all been given in terms of possibilities in general, i.e., logical possibilities. But for each definition, we could restrict the possibilities we are talking about and get restricted versions of these definitions. For example, physically necessary truths are those that hold in all the possibilities in which the physical laws hold. Morally necessary truths are those that are true in all the possibilities with the same moral code, etc. A set of propositions would physically entail another proposition if there is no physical possibility in which the propositions in the set are true and the other proposition is false. Two propositions are morally contradictory if there is no moral possibility in which both are true or both are false.

Even though explicit talk of these restricted kinds of logical concepts is rare, the theoretical apparatus is available and useful for trying to get clear on what various reasoners or arguers are in fact claiming. For example, in common discourse, when someone says that A entails B, I suspect they rarely mean that there is no possibility whatsoever in which A obtains and B does not; rather, for some contextually determined (though usually unspecified) group of possibilities there is no possibility in which A obtains and B does not. Similarly, for necessary and sufficient conditions; when someone says that snow is necessary for skiing, they probably do not mean that there is no possibility whatsoever in which there is skiing but no snow (there are in fact numerous possibilities—water skiing, roller skiing, sand skiing, skiing on

artificial pellets, etc.), but rather that our typical conception of skiing requires snow. In the sciences, they are rarely concerned with logically necessary and sufficient conditions, but rather with causally necessary and sufficient conditions—conditions that require or guarantee something else in all the possibilities consistent with the causal laws. The moral for critical thinking is that even when one encounters terms such as ‘entails’ or ‘contradictory’ or ‘necessary condition’ they may not be being used in their strictly logical sense, but rather being used over a subset of relevant possibilities.

### 3. Logic and the activity of reasoning

#### 3.1 *Logic and reasoning*

I conclude with some final comments about the application, and misapplication, of logical concepts in the study of reasoning. Logical systems are models. In particular, they are models of consequence relations between propositions. Some of the models are quite limited. For example, standard sentential or propositional logic systems ignore the internal structure of simple propositions and focus solely on connectives such as ‘and’, ‘or’ or ‘if,...then’. Others add elements to model ‘must’ and ‘can’ while still ignoring everything else, and so on. The hope is to ultimately get a model, or group of models, that illuminates the standards of good reasoning, at least with regards to inferential support. Like most models, logical models can be very helpful when properly applied within the domain they model. Trying to use the model outside the proper domain, however, can have drastic consequences. For example, claiming that the standard sentential logic system is a good model for explaining instances of good reasoning utilizing modal claims is clearly a mistake. (This is true not just for logical models. For example, using the “model” of the north star as a fixed point is extremely useful for general terrestrial navigation, but using the same model for routing certain sorts of messages, which requires quite precise location determination, gets poor results.) Similarly, since logic focuses on support relations and good reasoning usually involves not just adequate support, but good reasons as well, it is a mistake to think logic is the whole story of good reasoning. Indeed, logic has little to nothing to say about what makes reasons good reasons, but rather focuses on what can legitimately be inferred from whatever good reasons we find.

#### 3.2 *Arguments and explanations*

Clearly the target domain we are trying to understand and improve—the activities of reasoning, arguing, justifying, persuading, etc., are much more

complicated than any of the various logical systems that logicians produce to model certain aspects of those activities. And yet many theorists still try to find distinctions in the models that are really only distinctions in the activities and not really the concern of logic at all. For example, logicians and argumentation theorists have spent a lot of time trying to distinguish arguments from explanations. But suppose I lay out several reasons (including some reasons about what I think will happen in the next six months) why you should believe a particular company will fail in the next six months. Six months go by and the company fails and someone else asks “Why?” and I trot out my reasons again. Nothing has changed about the propositions involved, so, from the perspective of logic, there is one object, one set of propositions, here. Yet, how that object has been used has changed. Initially the reasons are used to argue that the company will fail. After the fact, the reasons are used to explain the company’s failure. We argue for propositions we are not sure of (or to convince others of propositions they are not sure of), but we explain propositions we are sure of, some of which may have been proved to us by argument, in order to understand *why* they are true. [Note that unlike my example, there are plenty of cases where the reasons one might give to argue for a proposition, which turns out to be true, need not be the reasons given when explaining why the proposition is true. For example, if something unexpected happens in the six months that contributes to the company’s failure that is likely to be a part of the act of explaining even though it was not part of the act of arguing.] The fact that there is a difference between acts of arguing and acts of explaining does not mean that, in the domain of logic, we should find separate kinds of things—arguments on the one hand and explanations on the other.

### 3.3 *Inferring and implying*

Going in the other direction, no one doubts that, considered in terms of propositions and support relations the inference from A to B and the implication of B by A are the same thing. But it is a mistake to think that the act of inferring is the same as the act of implying. You assert a group of facts (with the intention that I draw conclusions from those facts). I, being a good reasoner, draw those conclusions. You *imply* those conclusions and I *infer* those conclusions. Put another way, if I ask someone what they are inferring, I am asking about reasoning going on in their head, but if I ask someone what they are implying, I am asking about reasoning they hope to be going on in other people’s heads. Put yet another way, reasons do not infer conclusions, but rather imply them. People, when considering those reasons on their own, infer those conclusions, but do not imply them.

### 3.4 *Deductive and inductive*

Sometimes concepts are misapplied in both the model and the target domain. For example, some logic textbooks and critical thinking textbooks try to distinguish deductive arguments from inductive arguments, but from the perspective of logic there is nothing about the sets of propositions that compose arguments that make one kind of set deductive and another set inductive. For every group of reasons and a given conclusion we can ask whether the reasons entail the conclusion or not (the domain of deductive logic) or whether those very same reasons offer some support weaker than entailment or not (the domain of inductive logic.) Nor is it clear that we reason deductively or inductively—when we reason, we infer one or more propositions from others. Of that reasoning we did, we might wonder whether it is good or bad. The answer to that question will, in part, depend on what counts as good enough support in the situation in which I am using the reasoning. If the context requires entailment and the reasons do entail the conclusion, then the reasoning is adequate with regards to its support relation. If the reasons do not entail the conclusion, then it will fail to be adequate in such a situation. Similarly for a required support relation weaker than entailment—if reasons support the conclusion at or above the required level, then the support relation is adequate, whereas if it is below the required level the support relation is not adequate. The reasoning is one act of reasoning—whether the actual support relation of that reasoning is adequate or not depends on the situation. But none of this suddenly makes it the case that there are two distinct kinds of reasoning going on (even if there is a felt difference between realizing some reasons entail a conclusion versus realizing some premises only strongly support a conclusion.)

### 3.5 *Linked vs. convergent arguments*

One final example. The push for general principles often takes something that may track a real distinction or property in a certain specific set of cases and try to generalize it to all cases. For example, there is a strong intuition that reasons such as: “If you pass the test, then you will pass the course” and “You pass the test” work together to support the conclusion “you will pass the course” whereas reasons such as “You read all the supplemental material” and “You took good notes” and “You went to the tutor consistently” independently support the conclusion that “You are prepared for the test.” This intuition is strong enough, that numerous textbooks, especially those that use argument diagramming as a tool, try to distinguish arguments with linked premise structures from arguments with convergent premise structures. The

problem here is two-fold. On the one hand, attempts to actually provide a rule for determining when a set (or subset) of reasons are linked or not have, to date, all failed, at least if we trust the intuitions that generated the drive to generalize the phenomena in the first place. On the other hand, the underlying judgments of whether premises are working together or are independent seem to vary from person to person and context to context enough to suspect that the distinction may not be tracking a real phenomenon that deserves to be represented or captured in our logical models.

#### 4. Last word

Despite these injunctions to take care with the proper application of various concepts that have made their way into various textbooks, the core logical concepts of Section 2, such as *sufficient support* or *consistency* or *necessary condition* are useful in any study of reasoning. Even if good reasoners need to be careful and work diligently to determine which propositions are being expressed, and which possibilities are relevant, and what the needed standard of sufficient support is in a given situation, once these tasks are accomplished, we can evaluate our reasoning for inconsistencies and determine whether our reasons entail or at least adequately support our conclusions.

About the author:

**G.C. Goddu** has published extensively on the foundations of argumentation theory for the past twenty years. He is on the editorial board of the journal *Informal Logic* and is a two-time winner (2007, 2010) of the Association for Informal Logic and Critical Thinking's annual essay prize. He is currently the James Thomas Professor of Philosophy at the University of Richmond.