CHAPTER 4.

THE PROBLEM OF MISSING PREMISSES

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Abstract: Theorists of argument suppose that arguments with definite conclusions that do not follow logically from their premiss or premisses have a "gap-filling" unexpressed premiss, whose identification and addition to the stated premiss or premisses would produce an argument whose conclusion does follow logically. A common explanation for the omission of a premiss, found from Aristotle to Quine and Copi, is that arguers leave unstated known information that the readers or hearers can supply for themselves. Traditional Aristotelian logic developed a method for supplying the supposedly omitted premiss in the case of incomplete categorical syllogisms. This traditional approach has two weaknesses. The first weakness is that not every argument that is supposed to have a gap-filling unstated premiss is an incomplete categorical syllogism. This weakness can be remedied by recognizing that filling out an incomplete categorical syllogism by adding the appropriate categorical statement is a special case of constructing a covering generalization of the argument. The second weakness is that there is indeterminacy about what covering generalization to supply, with respect to both which repeated components of the argument are to be subject to generalization and how broadly to generalize over them. This weakness can be remedied by adopting a policy of maximum generalization, subject to constraints of context and plausibility. A more fundamental objection to this approach is phenomenologi-

cal: people reasoning and arguing in ways that are not logically compelling have no awareness of having omitted a premiss, even when they are reasoning something out for themselves. The whole approach of postulating an unexpressed gap-filler rests on a mistake, the mistake of supposing that there is a gap. Rather, logical consequence is a special case of a broader concept of consequence that includes material as well as formal consequence. The question to be asked in evaluating an argument with a definite conclusion is not how to expand it so as to make the conclusion follow logically but whether it has a non-trivially acceptable covering generalization that supports counterfactual instances. The broader concept of consequence has been recognized by Bolzano, Peirce, Ryle, Sellars, Toulmin, George, Brandom and others, but has not yet been recognized in introductory logic textbooks. It needs to be.

1. INTRODUCTION

At the First International Symposium on Informal Logic, held in Windsor, Ontario in June 1978, Ralph H. Johnson and J. Anthony Blair outlined a research agenda (Johnson & Blair 1980, pp. 25-26) for the sub-discipline of philosophy which their conference made newly self-conscious, a sub-discipline subsequently recognized by the International Federation of Philosophical Societies (FISP) under the title "philosophy of argumentation".[1] In their "unclassified and partial list of problems and issues in informal logic" (Johnson & Blair 1980, p. 25), there appeared what they called "the problem of assumptions and missing premises", which they characterized by a set of questions:

> What exactly is a missing premise? What different kinds of assumptions can be distinguished in argumentation? Which are significant for argument evaluation? How are missing premises to be identified and formulated? Are these just practical and pedagogical questions, or theoretical as well? (Johnson & Blair 1980, p. 25)

1. In the first circular for the FISP-sponsored World Congress of Philosophy in 2018, philosophy of argumentation is the 57th of 99 alphabetically ordered sections to which one could contribute papers (https://www.fisp.org/documents/WCP%202018%20First%20Circular%20English.pdf; accessed 2017 12 06).

Subsequently Ennis (1982) distinguished three types of implicit assumptions in arguments: backups implicitly assumed as support for a stated premiss, presuppositions without which a premiss of the argument would not make sense, and gap-fillers needed by the argument or implicitly used by the arguer to make the conclusion follow logically from the stated premiss or premisses.

The postulation of gap-fillers arises quite naturally from paying attention to real arguments.[2] Almost always, the conclusion of a real argument is not a logical consequence of its premiss or premisses.[3] However, if the stated premiss or premisses and the

2. By real arguments, I mean arguments that people produce in their efforts to justify their claims or to explain why they hold the opinions that they do or to point out the consequences of others' positions or to accomplish any other communicative purpose (except serving as an example in a logic textbook or as part of a Socratic question-and-answer refutation).

3. The point holds on any accepted conception of logical consequence, whether information-theoretic, modal, model-theoretic, set-theoretic, substitution-theoretic, schematic, speech-act-theoretic, or syntactic. An information-theoretic conception (Corcoran 1998) takes a proposition c to be a consequence of a set ∋ of propositions if and only if the information in the propositions in ∋ includes the information in the proposition p. A modal interpretation (Bradley and Swartz 1979; Etchemendy 1990) takes a proposition c to be a consequence of a set ∋ of propositions if and only if p is true in every possible "world" (i.e. state of affairs) in which the propositions in ∋ are true. A model-theoretic conception (Tarski 2002/1936) takes a sentence c in a formal (or formalized) language to be a consequence of a set ∋ of sentences in that language if and only if c is true on each uniform interpretation (or re-interpretation) of the extra-logical constants in c and in the sentences in ∋ on which every sentence in ∋ is true. A substitutional conception (Quine 1970) takes a sentence c in a formal or formalized language to be a consequence of a set ∋ of sentences in that language if and only if the sentence obtained from c as the result of any uniform substitution on the extra-logical constants in c and the sentences in ∋ is true if every sentence obtained from the sentences in ∋ by this substitution is true. A schematic conception (Quine 1972) takes a sentence c in a formal or formalized language to be a consequence of a set ∋ of sentences in that language if and only if c and ∋ are instances of at least one set of schemata c(x1, …, xn) and ∋(x1, …, xn) containing no extra-logical constants for which every instance of c(x1, …, xn) is true for which the corresponding instances of the sentence schemata in ∋(x1, …, xn) are true. A speech-act conception (Kearns 1997) takes a statement or propositional act c to follow from a set ∋ of statements or propositional acts if and only if a

stated conclusion are not hedged by such qualifiers as 'perhaps' or 'generally' or 'may', the conclusion will typically be a logical consequence of an expansion of the argument in which a premiss is added. Thus the notion arose that such arguments have an unexpressed premiss, variously described as missing (Johnson & Blair 1980), tacit (Hitchcock 1983), hidden (Gough & Tindale 1985), or suppressed (Copi & Cohen 2002).

2. THE TRADITIONAL APPROACH

The failure of most real arguments to conform to logicians' conceptions of the consequence relation has been recognized since the time of Aristotle, who found this failure in the speeches of orators and explained it as due to their accommodation of the limited attention span of their audience, for whom arguments needed to be concise and should therefore omit components that the audience could supply for themselves:

> An enthymeme is a rhetorical proof… The enthymeme must be a syllogism, with few <premisses>, often fewer than the primary syllogism. For, if any of them is familiar, it is not necessary to state it, for the hearer himself adds it. For example, <to show> that Doreius has won a crowned contest, it is sufficient to say that he has won at the Olympics, and it is not necessary to add that the Olympics are crowned, for everybody knows <that>. (Aristotle, *Rhetoric* I.1.1355a6, I.2.1357a16-20; my translation[4])

person is committed to accepting c who accepts the statements or propositional acts in ∋. A syntactic conception (Tarski 2002/1936) takes a sentence c in a formal or formalized language to be a consequence of a set ∋ of sentences in that language if and only if c is deducible from ∋ using the rules of inference of a sound logic for that language. There are variants on these conceptions. They are not equivalent to one another. In particular, application of the conceptions defined in terms of a formal or formalized language requires "translation lore" whose use requires judgment and can be quite complicated. The differences among the conceptions turn out to be relevant to the task of extending the concept of logical consequence to cover so-called "material consequence" (Sellars 1953).

4. *"esti d'apodeixis rhêtorikê enthymêma ... anankaion ... to d' enthymêma syllogismon, kai ex oligôn te kai pollakis elattonôn ê ho prôtos syllogismos· ean gar êi ti toutôn gnôrimon, oude dei legein· autos gar touto prostithêsin ho akroatês, hoion hoti Dôreius stephanitên agôna*

By the "primary syllogism", Aristotle here means the syllogism found in attempts like that of Socrates in Plato's "peirastic" (testing) dialogues to refute his interlocutors' theses. It is the kind of reasoning for which Aristotle subsequently developed the first system of formal logic, his categorical syllogistic. By definition, the conclusion of a syllogism follows necessarily from its premisses.[5]

Aristotle used the word 'enthymeme' (Greek '*enthymêma*') for the syllogism's rhetorical counterpart, which he took to be characterized by reasoning from likelihoods or signs (Aristotle, *Prior Analytics* II.27.70a10[6]). For Aristotle, it was not a defining feature of an enthymeme that it has fewer premisses than a "primary syllogism", only a frequent occurrence. Stoic logicians, however, defined an enthymeme as an incomplete syllogism.[7] Their definition became accepted in the European logical tradition. With syllogisms taken to be Aristotelian categorical syllogisms, with two premisses and a conclusion each of subject-predicate form and of a definite quality (affirmative or negative) and quantity (universal, particular or singular), the textbook tradition distinguished three types of enthymemes: first-order enthymemes in which the major premiss (the premiss containing the predicate of the conclusion) was missing, second-order enthymemes in which the minor premiss (the premiss containing the subject of the con-

nenikêken· hikanon gar eipein hoti Olympia nenikêken, to d' hoti staphantês ta Olympia oude dei prostheinai· gignôskousi gar pantes" (OCT text, ed. W. D. Ross)

5. "A syllogism is an argument in which certain things are posited and something other than the things laid down results of necessity through the things laid down" (*Topics* I.1.100a25-27, my translation). "*Esti dê ho syllogismos logos en hôi tethentôn tinôn heteron ti tôn keimenôn ex anankês symbainei dia tôn keimenôn*" (OCT text, ed. Ross). Similar definitions can be found in Aristotle's *Sophistical Refutations* (164b27-165a2) and in his *Prior Analytics* (I.1.24b18-20)

6. "An enthymeme is a syllogism from likelihoods or signs" (Aristotle, *Prior Analytics* II.27.70a10, my translation). "*Enthymêma de esti syllogismos ex eikotôn ê sêmeiôn*" (OCT text, ed. Ross).

7. "The enthymeme is an incomplete syllogism" (Epictetus, *Enchiridion* I.8.3; my translation). "*... atelês syllogismos esti to enthymêma*" (Teubner text, ed. Schenkl).

clusion) was missing, and third-order enthymemes in which the conclusion was missing (Copi & Cohen 2002, p. 270). The limited number of types of two-premiss categorical syllogisms made it possible to construct a sound and complete system for filling out incomplete categorical syllogisms of the three orders, helped by such generalizations from Aristotle's results as the principles that the middle term shared by both premisses must be "distributed" at least once, that a universal conclusion follows only from universal premisses, that a negative conclusion follows only if there is exactly one negative premiss, and so forth.

Consider, for example, the argument that birds are reptiles, because they are vertebrates, are suspended in a membrane in their embryonic stage, and are descended from the most recent ancestor of living turtles, crocodilians and lizards. The conclusion is a universal affirmative statement whose subject is the term 'birds' and the premiss is another universal affirmative statement with the same subject. To make this argument into a categorical syllogism, one needs to supply a second premiss that links the predicate of the stated premiss to the predicate of the conclusion. The only categorical statement that does so in a way that produces a categorical syllogism is a universal affirmative statement whose subject is the predicate of the stated premiss and whose predicate is the predicate of the conclusion: All vertebrates that are suspended in a membrane in their embryonic stage and are descended from the most recent ancestor of living turtles, crocodilians and lizards are reptiles. The same method can be used to find a statement whose addition as a premiss or a conclusion will transform any incomplete categorical syllogism into a complete one.

Although Aristotle located the supposed omission of a premiss in the speeches of orators, samplings of arguments in scholarly books and in calls to radio talk shows have found a similar failure to conform to logicians' models (Hitchcock 2002; 2009). In fewer than 10% of the arguments discovered in either context (6% of

the scholarly arguments, 7.7% of the spoken arguments) was the conclusion a logical consequence of the premiss or premisses.

Aristotle's explanation of this phenomenon continues to be repeated, both in theoretical works (e.g. Quine 1972, p. 169) and in introductory logic texts (e.g. Copi & Cohen 2002, p. 269). Acceptance of the explanation that a premiss has been omitted from such arguments naturally raises the question of how one is to discover the premiss that has been omitted, especially in circumstances like the analysis of a written or recorded argument, where one cannot ask the author to supply it or to assent to one's suggestion as to what it was.

3. FIRST WEAKNESS: LIMITED SCOPE

The traditional approach of turning an incomplete categorical syllogism into a complete one has at least two weaknesses. First, not every argument whose conclusion is not a logical consequence of its premiss or premisses is an incomplete categorical syllogism. We can ignore arguments whose conclusion is not a statement or is qualified by a word like 'probably' or 'presumably' or 'perhaps' or 'possibly', since there is no statement that can be added to such arguments as an additional premiss to make the conclusion a logical consequence of the expanded argument's premisses. Even setting aside such arguments, we can find arguments whose conclusion is not a logical consequence of their premiss or premisses but which are not incomplete categorical syllogisms. Consider for example the following argument, put forward by a woman caller to a radio phone-in program discussing the wish of a married woman to go to dance clubs without her husband:

> I think the reason why the wife wants to go to clubs is because she would prefer to feel younger again. You know, when you go to clubs, you know that you have it when you can pick up guys and stuff. I think she wants that, and the husband either doesn't have the need or he doesn't have it. So I think he should give her a break

and if he doesn't enjoy going give her that one night with the girls (Hitchcock 2009, "Appendix", p. 3).

This is a complex argument, which requires some analysis to tease out its structure. The ultimate conclusion, introduced by the word 'so', is supported immediately by the statement that immediately precedes it. This supporting statement is a conjunction, whose first conjunct repeats in somewhat different words the idea of the initial statement, for which the second statement in the paragraph is offered in support. Filling in anaphoric references, deleting the framing introductory phrases "I think" and "you know", construing "it" as referring to sex appeal, and using a standard numbering system, one might analyze the argument as follows:

1.1 When you go to clubs, you know that you have sex appeal when you can pick up guys and stuff.

1. The reason why the wife wants to go to clubs is because she would prefer to feel younger again. (The wife who wants to go to clubs wants to know that she has sex appeal.)

2. The husband either doesn't have the need to know that he has sex appeal or doesn't have sex appeal.

C. The husband should give his wife a break and if he doesn't enjoy going to clubs give her that one night out going to clubs with the girls.

The main argument from premisses 1 and 2 to conclusion C would be extremely difficult to massage into the form of an incomplete categorical syllogism. Nevertheless, its conclusion is an unqualified statement that does not follow logically from its premisses. To fit the argument into the logician's model, one needs some way of attributing an unstated premiss to it. But the approach of treating it as an incomplete categorical syllogism does not provide such a way.

The limited applicability of the traditional approach can be overcome by recognizing that filling out a first-order or second-order incomplete categorical syllogism is a special case of a more general procedure. It can be shown by complete enumerative induction that the premiss generated by the traditional approach is logically equivalent to a covering generalization of the argument with respect to the term shared by the stated premiss and the conclusion. We can use Aristotle's Doreius argument as an example, taking 'Doreius', the grammatical subject of both the premiss and the conclusion, as the shared term. To construct a covering generalization of such a one-premiss argument, we form a conditional statement with the premiss as the antecedent and the conclusion as the consequent:

> If Doreius has won at the Olympics, then Doreius has won a crowned contest.

We then replace the shared term with a variable of the appropriate type, in this case a variable ranging over individuals:

> If x has won at the Olympics, then x has won a crowned contest.

Formally, there should be an initial quantifier 'for every x', but we take the universal quantification to be conveyed by the use, borrowed from algebra, of small letters from the end of the alphabet as being implicitly universally generalized. We then transform the statement into something logically equivalent but more intelligible:

> Everyone who has won at the Olympics has won a crowned contest.

This is precisely the assumption that the traditional approach would supply. It can be shown by complete enumerative induction on the moods of the categorical syllogistic that the covering generalization of a first-order or second-order incomplete categorical syllogism with respect to the term shared by its premiss and its conclusion is logically equivalent to the statement whose

addition as a premiss would transform the argument into a complete categorical syllogism.

Let us then apply the more general approach to the main argument of the dance clubs example. To construct a covering generalization, one forms first of all the argument's associated conditional, i.e. the (material) conditional whose antecedent is the conjunction of the argument's premisses and whose consequent is the argument's conclusion. In the above example, we get the following conditional associated with the main argument:

> If the reason why the wife wants to go to clubs is because she wants to know that she has sex appeal and the husband either doesn't have the need to know that he has sex appeal or he doesn't have sex appeal, then the husband should give his wife a break and if he doesn't enjoy going to clubs give her that one night out going to clubs with the girls.

Next, one identifies the repeated content expressions in the conditional, making sure to include at least one content expression shared by the antecedent and the consequent but also noting content expressions repeated in the antecedent but not occurring in the consequent.[8] By a content expression I mean a word or phrase that can be replaced by a single independently significant word without loss of grammaticality (Hitchcock 1985, p. 84). In Aristotle's example, the phrase 'Doreius has won' is a content expression, because it can be replaced by the independently significant word 'win' without loss of grammaticality. In our current example, the repeated content expressions in the argument's associated conditional are "the wife", "go to clubs", "has sex appeal", and "the husband". Putting variables of the appropriate type in place of these phrases, and assuming introductory uni-

8. The reason for doing so anticipates the ultimate status of such a covering generalization as an inference-license rather than a premiss. Without the requirement of an overlapping variable, there is no real inference to license. A conclusion could be said to be a "consequence" of a set of premisses merely because it was true or merely because the premisses were not all true. See (Hitchcock 1998, pp. 24-27).

versal generalizations over these variables, one gets the following (simplified) generalization of the associated conditional:

> If x wants to do F because x wants to know that x has G and y either does not want to know that y has G or does not have G, then if y does not enjoy doing F y should let x do F.

Finally, one transforms the generalization into something more intelligible. In our example, the result might be a statement like the following:

> One person should let another person do what they want to do if the second person has a reason for doing it that the first person does not share.

The reader may think that this statement is too general as a candidate for the missing premiss of the dance clubs argument. We will consider this objection in due course.

4. SECOND WEAKNESS: INDETERMINACY

Extending the scope of the traditional approach in this way addresses its first weakness, that not every logically incomplete argument is an incomplete categorical syllogism. But a second weakness remains, which as it happens both the Doreius argument and the dance clubs argument illustrate. Any argument whose conclusion is not a logical consequence of its premiss or premisses can be expanded in more than one way so as to make the conclusion a logical consequence of the premisses of the expanded argument.[9] Consider Aristotle's Doreius argument.

9. The added premiss must entail (i.e. logically imply) the original argument's associated material conditional. Otherwise, it would be consistent to assert the added premiss and to deny the associated conditional. But denial of a material conditional is logically equivalent to assertion of its antecedent and denial of its consequent. If the conditional is an argument's associated conditional, this amounts to assertion of the original argument's premiss(es) and denial of its conclusion, thus rejecting the claim that the conclusion follows from the premiss(es) along with the added premiss. Given that the associated conditional cannot coherently be supported by denial of

If we take the traditional approach to completing it, we obtain the unstated premiss that anyone who has won at the Olympics has won a crowned contest. This assumption follows logically from the assumption that Aristotle supposes will be supplied by the hearer, that the Olympics are a crowned contest. But it is logically weaker. For it would be true if, for example, the Olympics were not a crowned contest but it was a requirement for competing in the Olympics that one have previously won a crowned contest. (Here we use a modal conception of logical consequence to show that Aristotle's unstated premiss is not a logical consequence of the unstated premiss generated by the traditional approach to enthymemes.)

One can however transform Aristotle's example so that the traditional approach generates Aristotle's assumption. To do so, one needs to massage the stated components so as to make the entire repeated phrase "Doreius has won" into a term. One might for example rephrase the argument as follows:

> Some contest at the Olympics is a contest which Doreius has won; therefore, some contest which Doreius has won is a crowned contest.

With this rephrasing, the traditional approach generates the assumption that every contest at the Olympics is a crowned con-

its antecedent or affirmation of its consequent (on pain of inconsistency in the first case and begging the question in the second case), it needs a logically stronger statement to support it. The obvious candidates for such a logically stronger statement are universal generalizations of it. But any such generalizations can vary with respect to which extralogical components of the associated conditional are subject to generalization and how broad is the scope of the generalization. For many formal languages, in fact, including the languages of first-order classical and intuitionistic logic, it is a theorem (the Craig interpolation theorem) that, if one formula entails another, there is an intermediate formula such that the first entails the intermediate formula and the intermediate formula entails the second, an intermediate formula that contains all and only the extralogical symbols in the first and second formulas (Craig 1957, p. 267). Thus, if a supposedly logically incomplete argument is symbolized in such a language, there will be an alternative to the proposed missing premiss that is an interpolant between it and the argument's associated conditional.

test, which is the assumption that Aristotle supposed the hearer can supply.[10] Aristotle based his choice on what he assumed every hearer knows. But what of the many arguments that people encounter where they do not have the required background knowledge to select a known truth as the completion of a supposedly incomplete argument?

Aristotle's Doreius argument illustrates one way in which it can be indeterminate which unstated premiss to attribute to a supposedly incomplete argument: how much or how many of the repeated components of the argument are to be abstracted from (or generalized over) in constructing the unstated premiss. Do we abstract from (generalize over) 'Doreius' or over 'Doreius has won'? Another example of this sort of indeterminacy, used in (Hitchcock 1985), is the argument sometimes heard that marijuana should be legalized, because it is no more dangerous than alcohol, which is already legal. Here we have three repeated terms: 'marijuana', 'legal', 'alcohol'. If we generalize on all three terms, we get as an unstated assumption of the argument that anything that is no more dangerous than another thing should get whatever status that other thing has. But it seems unfair to attribute to the argument an assumption that generalizes over 'legal' and to object that driving a car is no more dangerous than cycling, which is already unlicensed, but that nobody would agree that driving a car should be unlicensed. It seems fairer to the argument not to treat 'legal' as a variable component.

The marijuana argument illustrates another way in which it can be indeterminate which unstated premiss to attribute to a supposedly incomplete argument: how broadly to generalize over a repeated component of the argument. Intuitively, it would be unfair to generalize so broadly over the repeated terms 'marijuana' and 'alcohol' as to expose the argument to the objection

10. For the proof that the traditional completion of the above incomplete categorical syllogism is logically equivalent to its covering generalization with respect to the shared term 'contest which Doreius has won', see the appendix.

that not wearing a seat belt is no more dangerous than hang gliding, which is already legal, but that it is not the case that not wearing a seat belt should be made legal. The unfairness illustrates a felt need to restrict the scope of the generalization over 'marijuana' and 'alcohol' to recreational drugs: Any recreational drug that is no more dangerous than a legal recreational drug should be legalized.

Such restrictions of the scope of a covering generalization apply particularly to what I came to call 'occasional arguments' (Hitchcock 2011). Quine (1960) used the term 'occasional sentence' to refer to a sentence whose truth-value is partly a function of the occasion of its utterance. Similarly, an occasional argument is an argument whose inferential scope is partly a function of the occasion of its utterance. The following is an example, provided by Robert Ennis in an e-mail communication:

> … when Michael Scriven and I were trying to find our way to Detroit airport in the car he rented, I said at one point, "The sign says 'Chicago' [to the right], so we should turn right there." (We were trying to get on I-94 going to the airport.) (e-mail communication, 2009 June 8)

The quoted argument, including Ennis's bracketed elaboration, has as a premiss that the sign says 'Chicago' to the right and as a conclusion that we should turn right "there", i.e. at the place where the sign points. The repeated content expressions in the argument are thus 'the sign' and '[to the] right'. The covering generalization with respect to these repeated content expressions is that we should turn in the direction indicated by any sign that says 'Chicago'. This generalization is a plausible candidate for an implicit premiss of Ennis's argument, with the caveat that it needs restriction to the situation in which Ennis advanced his argument. For example, after having gotten on I-94 going to the airport, at the exit ramp from I-94 to the airport, it would be a mistake for the driver to turn at that place in the direction indicated by a sign that said Chicago. The inferential scope of

Ennis's argument is indicated by the additional information that he supplied in his e-mail message. He and the driver were trying to get on I-94 going to Detroit airport. As one can confirm by consulting a road map, and as he and the driver both knew at the time, they were on I-96, having come across the Ambassador Bridge from Windsor, where they had been attending a conference. To restrict the scope of the covering generalization, it is necessary to supplement the premisses with the relevant contextually available information that is mutually known by the arguer and the addressee, thus producing the following expanded argument, displayed in a standard form:

1. We are on I-96 after having crossed the Ambassador Bridge.
2. We are trying to get on I-94 going to Detroit airport.
3. The sign says 'Chicago' to the right.

C. We should turn right where the sign says.

The supplementation of such occasional arguments has the function of specifying the scope of a plausible covering generalization. It does not make the conclusion a logical consequence of the expanded set of premisses. The repeated content expressions in the expanded argument are 'we', 'the sign', and 'right'. The covering generalization with respect to these expressions, when reformulated for intelligibility, is that anyone on I-96 after having crossed the Ambassador Bridge who is trying to get on I-94 going to Detroit airport should follow the signs that say 'Chicago'.

After this introduction of the concept of an occasional argument, we can return to the objection that the covering generalization supplied for the dance clubs argument was too general. The proposed covering generalization was that one person should let another person do what they want to do if the second person has a reason for doing it that the first person does not share. The argument, however, seemed very specific to the issue of what a husband should let his wife do. In the context, there is a

concern, pointed out explicitly by the talk-show host, that "most of them [dance clubs—DH] are pick-up joints". The caller's claim that the wife wants to go to dance clubs in order to know that she still has sex appeal is thus implicitly a denial that she wants to go to them in order to pick up a man with whom she will be unfaithful to her husband. She doesn't want to pick up a guy; she wants to know that she still *can* do so. The caller assumes that the wife would be willing for her husband to accompany her but that he is not interested in going to dance clubs, thus reinforcing the implicit denial that she wants to be unfaithful to her husband. With these aspects of the context identified, we can treat the dance clubs argument as an occasional argument and expand it somewhat as follows:

1. The reason why the wife wants to go to clubs is because she would prefer to feel younger again. (The wife who wants to go to clubs wants to know that she has sex appeal.)
2. The husband either doesn't have the need to know that he has sex appeal or doesn't have sex appeal.
3. The wife who wants to go to clubs does not want to be unfaithful to her husband.

C. The husband should give his wife a break and if he doesn't enjoy going to clubs give her that one night out going to clubs with the girls.

With this expansion of the argument, the most plausible covering generalization will restrict the scope of those who permit behaviour and those whose behaviour is permitted to husbands and wives respectively. Treating 'the wife', 'the husband' and 'go to clubs' as repeated content expressions subject to generalization, one gets the covering generalization that a husband whose wife wants to do something to know that she has sex appeal and not in order to be unfaithful to him should allow her to do it if he is not interested in accompanying her.

The Doreius argument, the marijuana argument and the dance clubs argument illustrate in various ways the indeterminacy of

the approach of constructing a covering generalization for a supposedly incomplete argument in order to make its conclusion follow logically from its premiss or premisses. If more than one content expression is repeated, it is indeterminate which of them is to be generalized over. If the variable that replaces a repeated content expression does not disappear in the simplification of the covering generalization, the scope of this variable (i.e. in a formal language the universe of discourse) is indeterminate. Features of the context may be supplied as additional implicit premisses in order to specify this scope.

To deal with this indeterminacy, I proposed in Hitchcock (1985, pp. 93-94) that one should attribute to an unhedged argument whose conclusion is not a logical consequence of its premiss or premisses the most general possible covering generalization that was plausible in the context. Specifically, subject to considerations of context and plausibility, one should generalize over the entirety of a repeated molecular content expression rather than over a proper part of it, over all distinct repeated content expressions, and over the entire category of items of the kind signified by a repeated content expression.

This approach goes as far as one can in rescuing the traditional approach to filling out with one or more missing premisses an unqualified argument whose conclusion is not a logical consequence of its premiss or premisses. It addresses the weaknesses of the limited scope of Aristotle's categorical syllogistic and the indeterminacy of the thesis that an argument of this sort implicitly assumes a covering generalization of the argument.

5. A FUNDAMENTAL OBJECTION

There remains, however, a fundamental objection to the claim that the assumption so supplied is an implicit premiss of the argument, left unstated because hearers or readers can supply it themselves. The objection is phenomenological. If we pay attention to our own mental processes when we are reasoning to our-

selves in this allegedly incomplete way, we have no awareness of having omitted a premiss. Further, it would be incoherent to suppose that we are leaving out a premiss because our intended audience can supply it, because we ourselves are the intended audience. Readers can recall for themselves a recent inference of the type discussed in this chapter, and can verify the absence of an unstated premiss in their thinking.

The whole tradition of supposing that reasoners and arguers leave unstated a premiss on which they are relying, I maintain, rests on a mistake (Hitchcock 1998). The mistake is to suppose that the only way that a conclusion can follow definitely from premisses is logically. Logical consequence is rather a special kind of consequence, distinguished by the absence of extra-logical terms in its articulation. Consequence in general can be characterized schematically or modally. Schematically, a conclusion follows definitely from a set of premisses if and only if the argument is of a form that rules out non-trivially, for both actual and counter-factual cases, that the premisses are true and the conclusion untrue (or, more generally, non-acceptable). The conclusion of Aristotle's Doreius argument follows in this way from its premiss, because it is of the form 'x has won at the Olympics, so x has won a crowned contest' and this form not only has no actual counter-examples but would not have counter-examples if others had won at the Olympics; further, the absence of counter-examples is not due to the absence of any instances with a true premiss or to the absence of any instances with an untrue conclusion (Hitchcock 2011). To determine whether the conclusion of an unqualified argument follows from its premiss or premisses, one needs to investigate whether it has such a form. The so-called implicit premiss is thus not a premiss, but the articulation in statement form of a possibly valid schema. If one's purpose in considering an argument is to determine whether its conclusion follows, constructing an implicit premiss is a superfluous spinning of wheels. It is more direct to seek

a counterfactual-supporting covering generalization that would license the inference from premisses to conclusion. For this purpose, it may be necessary to appeal to known features of the context of utterance of the argument that narrow the scope of the variables in the covering generalization, i.e. in a formal context to specify the universe or universes of discourse over which the variables range. Articulation of these features attributes one or more implicit premisses to the argument, and to this extent the implicit premiss tradition has some merit. But the point of such supplementation is not to make the conclusion a logical consequence of the supplemented set of premisses but to narrow the scope of the substantive covering generalization in virtue of which the conclusion follows.

6. MATERIAL CONSEQUENCE

Recognition of a broader conception of consequence than logical or formal consequence is not new. George (1983) finds it already in Bolzano's 1837 *Wissenschaftslehre* (Bolzano 1972/1837), in the form of a substitutional conception of consequence where not all the content expressions need be subject to substitution. Peirce (1955/1867-1902) recognized that people reason in accordance with, rather than from, what he called "leading principles". Ryle (1950) argued that a hypothetical statement like 'If today is Monday, tomorrow is Tuesday' is not a premiss of a corresponding argument like 'Today is Monday, so tomorrow is Tuesday", but rather the principle in accordance with which the conclusion of the argument is drawn. Sellars (1953) argued that there were not only formal rules of inference but also material rules of inference, which determined the meaning of descriptive terms; his student Richard Brandom has developed that idea in his "inferential semantics" (Brandom 1994; 2000). Toulmin (1958) influentially distinguished the "data" or "grounds" (Toulmin, Rieke & Janik 1978) on which arguers based their claims from the "warrants" that licensed the transition from grounds to claim and

pointed out that most warrants were substantive rather than analytic.

In my own work, I have developed a schematic conception of consequence that includes both formal and material consequence, and have extended this conception to defeasible inferences with a modally qualified conclusion. The end result of this development is the following statement:

> A conclusion follows from given premisses if and only if an acceptable counterfactual-supporting covering generalization of the argument rules out, either definitively or with some modal qualification, simultaneous acceptability of the premisses and non-acceptability of the conclusion, even though it does not rule out acceptability of the premisses and does not require acceptability of the conclusion independently of the premisses (Hitchcock 2011, p. 224).

Of two contemporary accounts of the truth-value of counterfactual statements, those of David Lewis (1973) and Judea Pearl (2009), Pearl's structural model semantics is easier to apply than Lewis's closest world semantics when determining the truth-value of a counterfactual instance of a covering generalization (Hitchcock 2014), and gives intuitively correct results. The consequence relation described in the above-quoted statement satisfies three of the five structural rules of consequence identified in (Gentzen 1964/1935)—namely, reflexivity, contraction and permutation. It satisfies restricted forms of the cut rule and the weakening rule (Hitchcock 2017, pp. 174-177). There is scope for further investigation of the expanded conception of consequence.

Despite these contributions, introductory textbooks continue to treat logical consequence as the only kind of definite consequence relation, and to give advice on filling out arguments whose conclusion is not a logical consequence of their premiss or premisses so as to make it a logical consequence of the expanded argument. Thus the main problem of missing premisses in contemporary logical education is the problem of failing to recog-

nize that in general no premiss is missing. In this respect, the logical tradition in its conservatism has not yet gotten beyond Aristotle's mistake.

APPENDIX

We can use the rephrasing of Aristotle's Doreius argument to provide another example of proving the logical equivalence of the traditionally supplied completion of a categorical syllogism to the covering generalization of the argument with respect to the term shared between premiss and conclusion. We need to prove that the covering generalization of the rephrased argument with respect to the term 'contest which Doreius has won' is logically equivalent to the statement that every contest at the Olympics is a crowned contest, which is the statement generated by the traditional approach to such an enthymeme. To do so, we infer each statement from the other. The following proof deduces the traditionally generated statement from the covering generalization:

1. For every F, if some contest at the Olympics is F, some F is a crowned contest. (covering generalization)
2. If some contest at the Olympics is a non-crowned contest, then some non-crowned contest is a crowned contest. (from 1, by instantiation)
3. But no non-crowned contest is a crowned contest. (logical truth)
4. Hence no contest at the Olympics is a non-crowned contest. (from 2 and 3, by *modus tollendo tollens*)
5. That is, every contest at the Olympics is a crowned contest. (from 4, paraphrasing)

Now we deduce the covering generalization from the traditionally generated statement:

1. Every contest at the Olympics is a crowned contest. (traditionally generated statement)
2. Suppose (for conditional proof) that some contest at the Olympics is F. (assumption)
3. Then some F is a crowned contest. (from 1 and 2, by existential quantifier elimination, universal quantifier elimination, conditional elimination, and existential quantifier introduction)
4. Hence, if some contest at the Olympics is F, then some F is a crowned contest. (from 2 and 3, by conditional introduction, discharging supposition 2)
5. Hence, for every F, if some contest at the Olympics is F, some F is a crowned contest. (from 4, by universal quantifier introduction)

The first half of the proof illustrates the need to be judicious in choosing one's instantiation of the covering generalization when deriving the traditionally supplied additional premiss.

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